Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 6, Issue 6; July-September, 2019, pp. 344-348

http://www.krishisanskriti.org/Publication.html

Creep Analysis in Anisotropic Disc in Presence of Thermal Gradients

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Abstract—*In this paper, emphasis has been done on development of analytical model capable of performing plastic stress and strain analyses for rotating an anisotropic containing varying amount of SiC particles in the radial direction. The thermal gradient experienced by the disc is the result of breaking action as estimated by Finite Element Method. The steady state creep behavior of disc is analyzed by Sherby's constitutive model and Hill's criteria for yielding. The creep response of rotating disc is expressed by a threshold stress with value of stress exponent as 8. The creep parameters characterizing difference in yield stresses have been used from the available experimental results in literature. It is concluded that for designing a rotating disc, the presence of thermal gradients needs attention from the point of view of steady state creep and the creep response in anisotropic discs operating under thermal gradients are observed to be significantly lower than those observed in disc without thermal gradients.*

Keywords: *Creep, Anisotropy, Thermal Gradients, Rotating disc, Composites.*

1. INTRODUCTION

Under many service conditions, rotating disc such as discs of gas turbines, jet engines, automotive and aerospace braking systems are usually operated at relatively higher angular velocities. Materials of discs are required to sustain steady loads for long period of time under different temperature conditions. In such conditions, material may continue to creep until its usefulness is seriously impaired. As a result, number of researchers has studied the creep behavior in composite materials with property of superior heat resistance. Pandey et al. [1992] studied the steady-state creep behavior of Al – SiCp composites under uniaxial loading condition in the temperature range between 623K and 723 K for different combinations of particle size and volume fraction of reinforcement and found that the composite with finer particle size has better creep resistance than that containing coarser ones. In Durodola and Attia [2000] investigated the potential benefits of using several forms of fiber gradation in FGM rotating discs using finite element method and direct numerical integration. It was observed that the different forms of property gradation modify the stress and displacement fields in FGM discs compared with uniformly reinforced discs.

Singh and Ray [2005] have studied creep in rotating discs of composite materials. The authors have estimated steady state creep response in a rotating isotropic FGM disc without thermal gradient using Norton's power law. It is concluded that, in a rotating isotropic FGM disc with linearly decreasing particle content from the inner to the outer radius, the steady state creep response in terms of strain rates is significantly superior compared to that in a disc with the same total particle content distributed uniformly.

Gupta et.al. [2004] have analyzed the creep behavior of a rotating disc having constant thickness and made of isotropic functionally graded material (FGM).

The effect of anisotropy on the stress and strain rates have been studied and concluded that the anisotropy of the material has a significant effect on the creep of a rotating disc [Chamoli et al., 2010].

Keeping this in mind, the study ends with an effort to determine the creep behavior for the particle reinforced anisotropic disc with constant thickness in presence of thermal gradients and compare it with those anisotropic disc with the operating under isothermal conditions. The material parameters of creep vary along the radial direction in the disc due to varying composition.

2. Finite Element Analysis of Thermal Gradient in a Composite Disc

The temperature gradient originating due to the braking action of the discs has been obtained by Finite Element Analysis. For this purpose, the disc with inner radius of 31.75 mm, outer radius 152.4 mm and thickness 5mm is supposed. The FGM disc was assumed to rotate with an initial rpm of 15,600, which is reduced to 15,000 rpm due to breaking action. An estimated heat flux of $130 kW/m^2$ has been applied over an annular area with inner radius 142.4 mm and outer radius 152.4 mm, while the remaining surfaces of the FGM disc have been exposed to ambient conditions with convective heat transfer coefficient of $25kW/m^2\,K$ and an ambient temperature of 303 K. For a particular ring, the thermal conductivity $K(r)$ is assumed to be constant and calculated using the rule of mixture as given below

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \quad (2.1)$$

where the matrix conductivity is $K_m = 247W/m\,K$ and the diserpersoid conductivity is $K_d = 100W/m\,K$. The temperature $T(r)$, obtained at any radius r is presented below in the form of regression equation as

$$T(r) = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5 \quad (2.2)$$

where the coefficients a_0, a_1, a_2, a_3, a_4 and a_5 for different disc are taken from Gupta et al..

3. Assumptions in Constant Thickness Disc

Consider an aluminum silicon-carbide particulate composite disc of constant thickness h having inner radius, a and outer radius, b rotating with angular velocity, ω (radian/sec). From symmetry considerations, principal stresses are in the radial, tangential and axial directions. For the purpose of analysis the following assumptions are made:

1. Stresses at radius of the disc remain constant with time i.e. steady state condition of stress is assumed.
2. Elastic deformations are small for the disc and can be neglected as compared to the creep deformations.
3. Biaxial state of stress ($\sigma_z = 0$) exists at any point of the disc.
4. Frictional shear stress induced due to braking action is estimated to be 10^{-5} MPa, which is very small compared to creep stresses and therefore, can be neglected.
5. The composite shows a steady state creep behavior which may be described by following Sherby's law [1977],

$$\dot{\bar{\varepsilon}} = [M(r)(\bar{\sigma} - \sigma_0(r))]^n \quad (3.1)$$

Where, $M(r) = \frac{1}{E}\left(\frac{AD_\lambda \lambda^3}{|b_r|^5}\right)^{1/n}$ is the creep parameter and $\dot{\bar{\varepsilon}}$, $\bar{\sigma}, n$, $\sigma_0(r), A$, D_λ, λ, b_r, E are the effective strain rate, effective stress, the stress exponent, threshold stress, a constant, lattice diffusivity, the sub grain size, the magnitude of burgers vector , Young's modulus.

The values of material parameters $M(r)$ and $\sigma_0(r)$ in terms of $P, T(r)$ and V have been obtained from the creep results by using the experimental results reported by Pandey et al., (1992) for Al-SiCp composite under uniaxial loading using the following regression equations,

$$M(r) = e^{-35.38} P^{0.2077} T(r)^{4.98} V^{-0.622} \quad (3.2)$$

$$\sigma_0(r) = -0.03507P + 0.01057T(r) + 1.00536 - 2.11916 \quad (3.3)$$

In a FGM disc, with the creep parameters $M(r)$ and $\sigma_0(r)$ will vary radially due to variations in temperature $T(r)$. In the present study, the particle size (P) and the particle content (V) are taken as $1.7\,\mu m$ and 20% over the entire disc. Thus, for a given FGM disc under known temperature both the creep parameters are functions of radial distance and their values $M(r)$ and $\sigma_0(r)$ at any radius (r), could be determined by substituting the values of particle size, particle content and temperature distributions into Eq. (3.2) and (3.3) respectively.

4. Mathematical Formulation

The generalized constitutive equations for creep in in an anisotropic composite disc under multiaxial stress takes the following form,

$$\dot{\varepsilon}_r = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\{(G+H)\sigma_r - H\sigma_\theta - G\sigma_z\} \quad (4.1)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\{(H+F)\sigma_\theta - F\sigma_z - H\sigma_r\} \quad (4.2)$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\{(F+G)\sigma_z - G\sigma_r - F\sigma_\theta\} \quad (4.3)$$

where F, G and H are anisotropic constants of the material. $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$, $\dot{\varepsilon}_z$ and σ_r, σ_θ, σ_z are the strain rates and the stresses respectively in the direction r, θ and z. $\dot{\bar{\varepsilon}}$ be the effective strain rate and $\bar{\sigma}$ be the effective stress. For biaxial state of stress $(\sigma_r, \sigma_\theta)$, the effective stress is,

$$\bar{\sigma} = \left\{\frac{1}{\left(\frac{G}{F}+\frac{H}{F}\right)}\left\{\sigma_\theta^2 + \frac{G}{F}\sigma_r^2 + \frac{H}{F}(\sigma_r - \sigma_\theta)^2\right\}\right\}^{1/2} \quad (4.4)$$

Using Eqs. (3.1) and (4.4), Eq. (4.1) can be rewritten as,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{\left[\left(\frac{G}{F}+\frac{H}{F}\right)x(r) - \frac{H}{F}\right][M(r)\,(\bar{\sigma} - \sigma_0(r))]^8}{\sqrt{\frac{G}{F}+\frac{H}{F}}\left[\left(\frac{G}{F}+\frac{H}{F}\right)x(r)^2 - 2\frac{H}{F}x(r) + \left(\frac{G}{F}+\frac{H}{F}\right)\right]^{1/2}} \quad (4.5)$$

Similarly from Eq. (4.2),

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{\left[\left(1+\frac{H}{F}\right) - \frac{H}{F}x(r)\right]\left[M(r)\,(\bar{\sigma} - \sigma_0(r)\right]^8}{\sqrt{\frac{G}{F}+\frac{H}{F}}\left[\left(\frac{H}{F}+\frac{G}{F}\right)x(r)^2 - 2\,\frac{H}{F}\,x(r) + \left(1+\frac{H}{F}\right)\right]^{1/2}} \tag{4.6}$$

$$\dot{\varepsilon}_z = -\left(\dot{\varepsilon}_r + \dot{\varepsilon}_\theta\right) \tag{4.7}$$

where, $x(r) = \frac{\sigma_r}{\sigma_\theta}$, is the ratio of radial and tangential stresses at any radius r.

Dividing Eq. (4.5) by Eq. (4.6), we get

$$\phi(r) = \frac{\left(\frac{G}{F}+\frac{H}{F}\right)x(r) - \frac{H}{F}}{\left(1+\frac{H}{F}\right) - \frac{H}{F}x(r)} \tag{4.8}$$

The equation of equilibrium for a rotating disc with varying thickness can be written as,

$$\frac{d}{dr}\left(r\,\sigma_r\right) - \sigma_\theta + \frac{\rho(r)\,\omega^2 r^2}{g} = 0 \tag{4.9}$$

where $\rho(r)$ is the density of FGM disc.

Boundary Conditions are

$\sigma_r(a) = 0 = \sigma_r(b)$ (4.10)

We get the tangential stress (σ_θ) from Eq. (4.9) by using Eq. (4.5) and Eq. (4.6),

$$\sigma_\theta = \frac{\psi_1(r)\left[A_0\ \sigma_{\theta_{avg}} - \int_a^b \psi_2(r).dr\right]}{M(r)\int_a^b \frac{\psi_1(r)}{M(r)}dr} + \psi_2(r) \tag{4.11}$$

where,

$$\psi_1(r) = \frac{\psi(r)}{\left\{\left(\frac{1}{\frac{G}{F}+\frac{H}{F}}\right)\left[\left(\frac{G}{F}+\frac{H}{F}\right)x(r)^2 - 2\frac{H}{F}x(r) + \left(1+\frac{H}{F}\right)\right]\right\}^{1/2}} \tag{4.12}$$

$$\psi_2(r) = \frac{\sigma_0(r)}{\left\{\left(\frac{1}{\frac{G}{F}+\frac{H}{F}}\right)\left[\left(\frac{G}{F}+\frac{H}{F}\right)x(r)^2 - 2\frac{H}{F}x(r) + \left(1+\frac{H}{F}\right)\right]\right\}^{1/2}} \tag{4.13}$$

and

$$\psi(r) = \left\{\frac{\sqrt{\frac{G}{F}+\frac{H}{F}}}{r}\cdot\frac{\left[\left(\frac{H}{F}+\frac{G}{F}\right)x(r)^2 - \frac{2Hx(r)}{F} + \left(1+\frac{H}{F}\right)\right]^{1/2}}{\left[\left(1+\frac{H}{F}\right) - \frac{H}{F}x(r)\right]}\exp.\int_a^r \frac{\varphi(r)dr}{r}\right\}^{1/8} \tag{4.14}$$

The average tangential stress may be defined as

$$\sigma_{\theta_{avg}} = \frac{1}{b-a}\int_a^b \sigma_\theta\,dr \tag{4.15}$$

Now $\sigma_r(r)$ can be obtained by integrating Eq. (4.8) within limits a to b,

$$\sigma_r(r) = \frac{1}{r}\left[\int_a^r \sigma_\theta\,dr - \frac{\omega^2\rho(r^3 - a^3)}{3}\right] \tag{4.16}$$

Thus, the tangential stress σ_θ and radial stress σ_r are determined by Eq. (4.11) and Eq. (4.16) respectively, for anisotropic disc with constant thickness. Then strain rates $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$ and $\dot{\varepsilon}_z$ calculated from equations (4.5), (4.6) and (4.7).

5. Numerical Computation

The stress distribution is evaluated from the above analysis by iterative numerical scheme of computation. For rapid convergence 75% of the value of $\sigma_\theta(r)$ obtained in the current iteration has been mixed with 25% of the value of $\sigma_\theta(r)$ obtained in the last iteration for the use in the next iteration

6. Results and Discussion

A computer program based on the analysis presented was developed and results obtained were validated with the experimental results by Wahl et. Al (1954) for the same type of disc. This comparison is shown in Figure 1. It is observed from this figure that there is good agreement between the results obtained from present analysis. The ratios anisotropic constants of a composite disc which has been taken in this study as $G/F = 1.34,\ H/F = 1.64$. The stress exponent and density of disc material have been taken as $n=8$ and $\rho = 2812.4\,kg/m^3$ respectively. The tangential stress operating under a thermal gradient is higher near the inner radius and lowers near the outer radius as compared to the disc without thermal gradients as shown in Figure 2.

The radial stress developing due to rotation in the anisotropic disc operating under a thermal gradient is higher over the entire radius as compared to the anisotropic disc without thermal gradients as shown as Figure 3. The effect of imposing any type of gradient separately or simultaneously is similar to that in tangential stress.

In Figure 4, the tangential strain rate decrease significantly over the entire radius in the anisotropic discs operating under the thermal gradients compared to the discs without thermal gradients. Although, the tangential stress in disc under the thermal gradients is higher near the inner radius than the disc without thermal gradients as shown in Figure 4, but lower operating temperature near the inner radius of the disc with thermal gradients is able to reduce the creep rate overcoming the effect of higher stress. Clearly, temperature near the inner radius and tangential stress near the outer radius of the disc with thermal gradients dominate the creep behavior when compared to those observed in the disc without thermal gradients under isothermal condition.

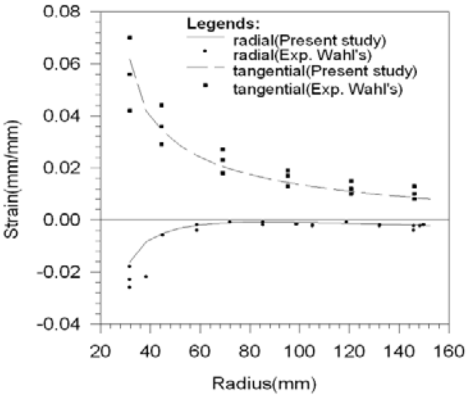

Figure 1
Comparison of theoretical (present study) and experimental strains in a rotating steel disc

In Figure 5, the effect of imposing thermal gradients on the radial strain rate in the anisotropic discs is similar to that observed for tangential strain rate. The magnitude of radial strain rate firstly increases rapidly with radial distance and then starts decreasing. It reaches a minimum before increasing again towards the outer radius in anisotropic discs in the presence/absence of thermal gradients.

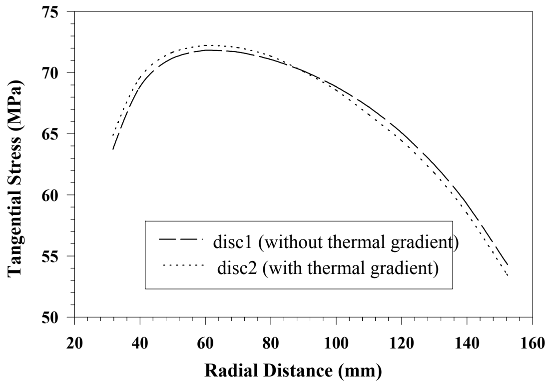

Figure 2
Variation of tangential stress along radial distance in composite discs with/without thermal gradient

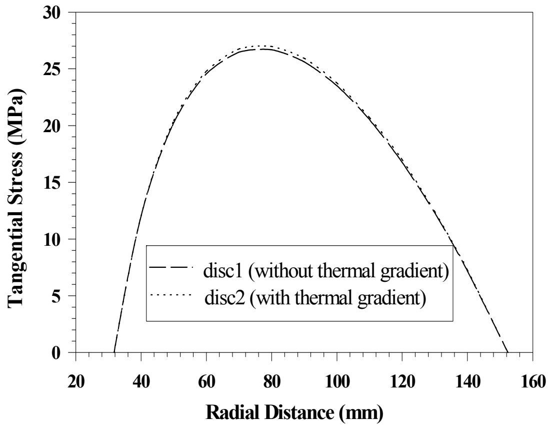

Figure 3
Variation of radial stress along radial distance in composite discs with/without thermal gradient

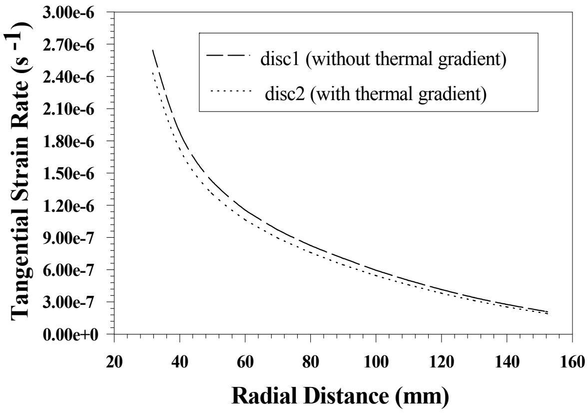

Figure 4

Variation of tangential strain rate along radial distance in composite discs with/without thermal gradient

CONCLUSION

From above discussion, it can be concluded that

1. The thermal gradients significantly affect the strain rate distribution in an anisotropic particle reinforced disc having constant thickness, but its effect on the distribution of stresses is relatively small.
2. Thermal gradient plays a significant role in developing the creep strains, it may be taken care of while design a rotating disc.

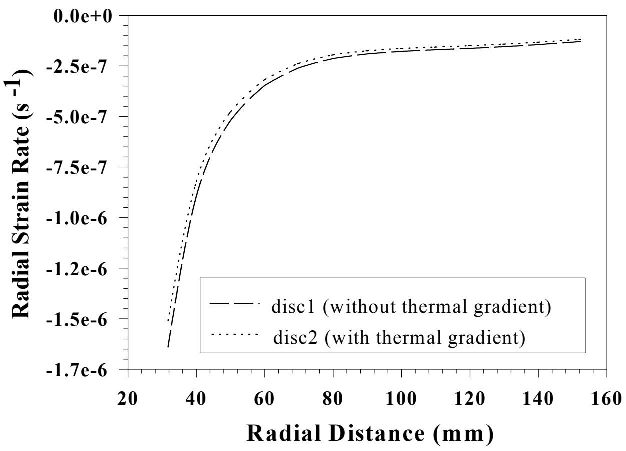

Figure 5

Variation of radial strain rate along radial distance in composite discs with/without thermal gradient

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